$$\langle a | f(b) \rangle = \langle f^*(a) | b \rangle$$

$$\langle f(c) | f(e) \rangle = \langle c | f^*(f(e)) \rangle$$

$$a = 0 \iff (\forall x \in E, \langle a | x \rangle = 0)$$

$$a = b \iff (\forall x \in E, \langle a | x \rangle = \langle b | x \rangle$$

2) $\ker(f^*) = (\mathrm{Im}(f))^{\perp}$?

$B = (e_1, \dots, e_n)$ bon de E.

$f \in \mathcal{L}(E)$.

$$\underset{B}{\mathrm{mat}}(f) = (a_{ij})_{1 \leq i, j \leq n}$$

$$a_{ij} = \langle ? | ? \rangle$$

$B = (e_1, \dots, e_n)$ $\underline{\text{bon}}$ de E.

$f \in \mathcal{L}(E)$.

$$\underset{B}{\operatorname{mat}}(f) = \begin{array}{c} \\ e_1 \\ \vdots \\ \vdots \\ e_n \end{array} \begin{array}{c} f(e_1) \;\dots\; f(e_j) \;\dots\; f(e_n) \\ \left(\begin{array}{ccc} & \boxed{\begin{array}{c} a_{1j} \\ \vdots \\ a_{nj} \end{array}} & \end{array} \right) \end{array}$$

$$f(e_j) = a_{1j} e_1 + \dots + a_{nj} e_n$$

$$a_{ij} \overset{?}{=} \langle ? \mid ? \rangle$$

$B=(e_1,\dots,e_n)$ $\underline{\underline{\text{bon}}}$ de E.

$f \in \mathcal{L}(E)$.

$$\underset{B}{\mathrm{mat}}(f) = \begin{array}{c} \\ e_1 \\ \vdots \\ e_n \end{array} \begin{array}{c} \begin{array}{ccccc} f(e_1) & \dots & f(e_j) & \dots & f(e_n) \end{array} \\ \begin{pmatrix} & & \boxed{\begin{matrix} a_{1j} \\ \vdots \\ a_{nj} \end{matrix}} & & \end{pmatrix} \end{array}$$

$$f(e_j) = a_{1j}e_1 + \dots + a_{nj}e_n$$

$$a_{ij} \overset{?}{=} \langle ? \mid ? \rangle$$

$\underline{\text{Rappel:}}$ $\boxed{\begin{array}{l} \text{Si } x = \sum_{i=1}^{n} x_i e_i \text{ et } (e_i) \text{ } \underline{\underline{\text{bon}}} \\ \text{Alors } x_i = \langle x \mid e_i \rangle \end{array}}$

$B = (e_1, \dots, e_n)$ bon de E.

$f \in \mathcal{L}(E)$.

$$\underset{B}{\mathrm{mat}}(f) = \begin{array}{c c} & \begin{array}{ccccc} f(e_1) & \dots & f(e_j) & \dots & f(e_n) \end{array} \\ \begin{array}{c} e_1 \\ \vdots \\ \vdots \\ e_n \end{array} & \left(\begin{array}{ccccc} & & a_{1j} & & \\ & & \vdots & & \\ & & \vdots & & \\ & & a_{nj} & & \end{array} \right) \end{array}$$

$$f(e_j) = a_{1j} e_1 + \boxed{a_{ij} e_i} + a_{nj} e_n$$

$$\triangle ! \quad a_{ij} = \langle f(e_j) \mid e_i \rangle$$

Rappel : Si $x = \sum_{i=1}^{n} x_i e_i$ et (e_i) bon

Alors $x_i = \langle x \mid e_i \rangle$

Notons :

$$\boxed{\begin{array}{l} \mathcal{B} = (e_1, \dots, e_n) \\ \underset{\mathcal{B}}{\mathrm{mat}}(f) = (a_{ij})_{1 \leq i,j \leq n} \\ \underset{\mathcal{B}}{\mathrm{mat}}(f^*) = (b_{ij})_{1 \leq i,j \leq n} \end{array}}$$

Soit $(i,j) \in [\![1,n]\!]^2$.

Il s'agit de montrer que $b_{ij} = a_{ji}$.

On a :

$b_{ij} =$ $\left(\underset{\mathcal{B}}{\mathrm{mat}}(f^*) = (b_{ij})_{1 \leq i,j \leq n}\right)$

$=$

$=$

$= a_{ji}$ $\left(\underset{\mathcal{B}}{\mathrm{mat}}(f) = (a_{ij})_{1 \leq i,j \leq n}\right)$