

$$\langle a | f(b) \rangle = \langle f^*(a) | b \rangle$$

$$\langle f(c) | f(e) \rangle = \langle c | f^*(f(e)) \rangle$$

$$a = 0 \Leftrightarrow (\forall b \in E, \langle a | b \rangle = 0)$$

$$a = b \Leftrightarrow (\forall c \in E, \langle a | c \rangle = \langle b | c \rangle)$$

$$2) \ker(\phi^*) = (\text{Im}(\phi))^\perp ?$$

$B = (e_1, \dots, e_n)$ bon $\mathbb{C} E$.

$f \in \mathcal{L}(E)$.

$\text{mat}_B(f) = (a_{ij})_{1 \leq i, j \leq n}$

$a_{ij} = \langle f(e_j), e_i \rangle$

$B = (e_1, \dots, e_n)$ basis of E .

$f \in \mathcal{L}(E)$. $f(e_1) \dots f(e_j) \dots f(e_n)$

$$\text{mat}_B(f) = \begin{pmatrix} e_1 \\ \vdots \\ e_n \end{pmatrix} \begin{pmatrix} a_{1j} \\ \vdots \\ a_{nj} \end{pmatrix}$$

$$f(e_j) = a_{1j}e_1 + \dots + a_{nj}e_n$$

$$a_{ij} = \langle ? | ? \rangle$$

$B = (e_1, \dots, e_n)$ bon $\hookrightarrow E$.

$f \in \mathcal{L}(E)$. $f(e_1) \dots f(e_j) \dots f(e_n)$

$$\text{mat}(f)_B = \begin{matrix} e_1 \\ \vdots \\ e_n \end{matrix} \left(\begin{array}{c} a_{1j} \\ \vdots \\ a_{nj} \end{array} \right)$$

$$f(e_j) = a_{1j}e_1 + \dots + a_{nj}e_n$$

$$a_{ij} = \langle ? | ? \rangle$$

Rappel: Si $x = \sum_{i=1}^n x_i e_i$ et (e_i) bon

$$\text{Alors } x_i = \langle x | e_i \rangle$$

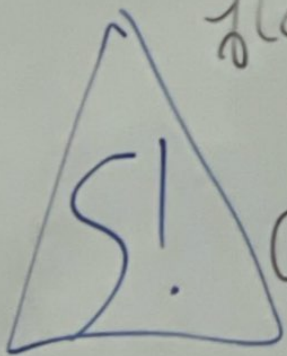
$B = (e_1, \dots, e_n)$ bon de E .

$f \in \mathcal{L}(E)$.

$f(e_1) \dots f(e_j) \dots f(e_n)$

$$\text{mat}_B(f) = \begin{pmatrix} e_1 \\ \vdots \\ e_n \end{pmatrix} \begin{pmatrix} a_{1j} \\ \vdots \\ a_{nj} \end{pmatrix}$$

$f(e_j) = a_{1j}e_1 + \boxed{a_{ij}e_i} + a_{nj}e_n$



$a_{ij} = \langle f(e_j) | e_i \rangle$

Rappel: $\forall x = \sum_{i=1}^n x_i e_i \quad \text{bon de } E$

Alors $x_i = \langle x | e_i \rangle$

Notons :

$$B = (e_1, \dots, e_n)$$

$$\text{mat}_B(f) = (a_{ij})_{1 \leq i, j \leq n}$$

$$\text{mat}_B(f^*) = (b_{ij})_{1 \leq i, j \leq n}$$

Soit $(i, j) \in \llbracket 1, n \rrbracket^2$.

Il s'agit de montrer que $b_{ij} = a_{ji}$.

On a :

$$b_{ij} =$$

=

=

$$= a_{ji}$$

$$\left(\text{mat}_B(f^*) = (b_{ij})_{1 \leq i, j \leq n} \right)$$

$$\left(\text{mat}_B(f) = (a_{ij})_{1 \leq i, j \leq n} \right)$$