Supp $|q| < 1$.

$$\sum_{n=5}^{+\infty} q^n = \frac{q^5}{1-q}$$

$$\sum_{n\in\mathbb{Z}} q^{|n|} = \sum_{n\in\mathbb{Z}^-} q^{|n|} + \sum_{n\in\mathbb{N}^*} q^{|n|}$$

$$= \sum_{n=0}^{+\infty} q^{|-n|} + \sum_{n=1}^{+\infty} q^{|n|}$$

$$= \sum_{n=0}^{+\infty} q^{n} + \sum_{n=1}^{+\infty} q^{n}$$

= [redacted] + [redacted]

Enfin : $\sum_{n\in\mathbb{Z}} q^{|n|} =$ [redacted] $\square$

$$\sum_{n\in\mathbb{Z}} q^{|n|} = \sum_{n\in\mathbb{Z}^-} q^{|n|} + \sum_{n\in\mathbb{N}^*} q^{|n|}$$

$$= \sum_{n=0}^{+\infty} q^{|-n|} + \sum_{n=1}^{+\infty} q^{|n|}$$

$$= \sum_{n=0}^{+\infty} q^{n} + \sum_{n=1}^{+\infty} q^{n}$$

$$= \frac{1}{1-q} + \frac{q}{1-q}$$

Enfin : $$\sum_{n\in\mathbb{Z}} q^{|n|} = \frac{1+q}{1-q}$$ $\square$

$$a \in \bigsqcup_{n \in \mathbb{N}} I_n \Longleftrightarrow \left( \exists n \in \mathbb{N},\ a \in I_n \right)$$

$$a \in \bigcap_{n \in \mathbb{N}} I_n \Longleftrightarrow \left( \forall n \in \mathbb{N},\ a \in I_n \right)$$

$$v_n \to \ell \iff v_n \sim \ell$$

où $\ell \neq 0$